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A. V. Ivlev and D. I. Zhukhovitskii

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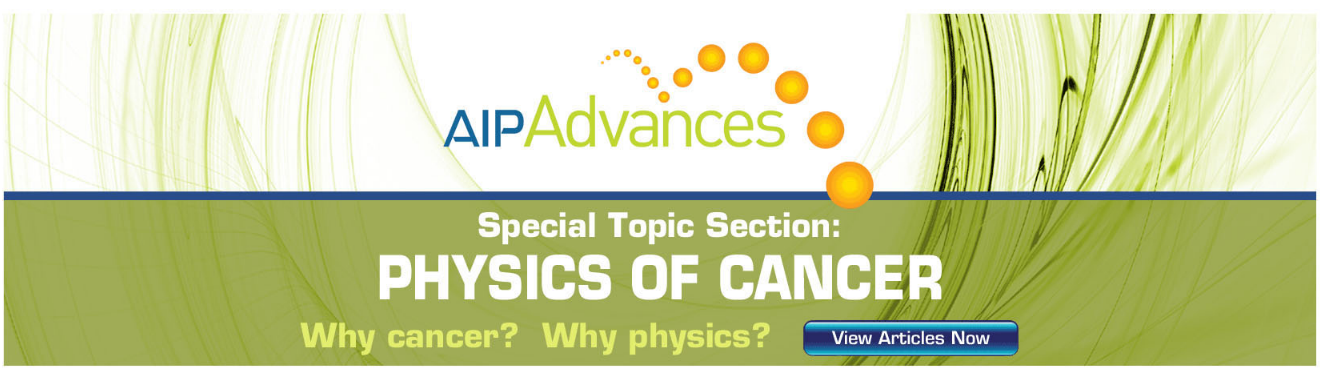

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The drag force on a subsonic projectile in a fluid complex plasma

A. V. Ivlev[1,a)] and D. I. Zhukhovitskii[2]
[1]*Max Planck Institute for Extraterrestrial Physics, 85741 Garching, Germany*
[2]*Joint Institute of High Temperatures, Russian Academy of Sciences, 125412 Moscow, Russia*

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The incompressible Navier-Stokes equation is employed to describe a subsonic particle flow induced in complex plasmas by a moving projectile. Drag forces acting on the projectile in different flow regimes are calculated. It is shown that, along with the regular neutral gas drag, there is an additional force exerted on the projectile due to dissipation in the surrounding particle fluid. This additional force provides significant contribution to the total drag. © *2012 American Institute of Physics*. [http://dx.doi.org/10.1063/1.4750070]

I. INTRODUCTION

Complex plasmas consist of monodisperse highly charged microparticles suspended in low-pressure weakly ionized plasmas, and represent a natural system which makes it possible to observe various collective phenomena at the level of individual particles.[1–9] Such particle-resolved studies are truly invaluable, in particular—because they allow us to identify limits of continuous approach and show where the discreteness of medium becomes important.

A broad variety of phenomena occurring in complex plasmas, e.g., waves,[10] hydrodynamic flows,[11–13] heat transport,[14] thermal convection,[15] etc., can normally be studied in the framework of continuous models. However, there are also problems where the discreteness of medium might be essential. The characteristic example here is the flow around a tracer particle—projectile—moving through a (two- or three-dimensional) complex plasma. Such projectiles are generated by using controlled mechanisms of acceleration,[16,17] or they can appear sporadically[18,19] (possible acceleration mechanisms in this case are still debated; e.g., in two-dimensional complex plasmas non-spherical grains can acquire energy due to the interaction with plasma flow[20]). Projectiles either lead to the formation of extended Mach cones/wakes, or produce localized disturbances of surrounding particles. Recently, it was suggested that the latter regime, realized when a relatively large subsonic projectile moves in a dense cloud of smaller particles, can be well approximated as a flow of an incompressible fluid.[21]

One of the possible ways to determine when the crossover from a continuous to discrete flow occurs in experiments with complex plasmas would be to observe emerging deviations from the continuous fluid behavior. Complex plasmas represent a class of complex fluids with the background friction exerted by the surrounding neutral gas.[6,7] Therefore, the corresponding continuous properties, such as the velocity distribution, flow patterns, and forces acting on a projectile, cannot be directly taken from textbooks but need to derived for a particular problem.

In this paper, we employ the incompressible Navier-Stokes equation to describe a subsonic particle flow induced in complex plasmas by a moving projectile. We calculate forces acting on the projectile in different flow regimes and show that, along with the regular neutral gas drag, there is an additional force exerted on the projectile due to dissipation in the surrounding particle fluid. Analysis of available experiments suggests that this additional force provides significant contribution to the total drag.

II. NON-VISCOUS FLOW

We start with considering a steady flow of non-viscous incompressible particle fluid around a projectile moving with the velocity $-\mathbf{u}$. In the reference frame of the projectile, the velocity $\mathbf{v}(\mathbf{r})$ and pressure $p(\mathbf{r})$ of the fluid are governed by

$$\nu(\mathbf{v}-\mathbf{u})+(\mathbf{v}\cdot\nabla)\mathbf{v}=-\frac{\nabla p}{\rho}, \tag{1}$$

$$\nabla\cdot\mathbf{v}=0, \tag{2}$$

where $\rho = mn$ $(=\text{const})$ is the fluid density expressed via the particle mass m and number density n, and ν $(\propto a^2/m)$ is the damping rate of individual particles (of the radius a) due to friction against the background neutral gas—the so-called Epstein drag.[6] The gas is assumed to be at rest (in the lab frame) and unaffected by the particle motion.[15]

As illustrated in Fig. 1, we treat the projectile (which is in fact a particle of the mass m_{p} and radius a_{p}) as a sphere of the effective radius $R \gg a_{\mathrm{p}}$ which is determined by the magnitude of the repulsion with surrounding particles. Such consideration is well justified when u is much smaller than the sound speed in the particle fluid: The coupling parameters of interaction between the projectile and particles as well as between particles themselves are large, so that the Wigner-Seitz cell model can be used, and then R is the sum of the Wigner-Seitz radii for the projectile and particle.[21]

We would like to stress that interactions between the projectile and particles are assumed to be *reciprocal* (e.g., electrostatic). Otherwise, when the action-reaction law is violated (e.g., for interactions mediated by ambient gas/plasma, such as thermophoretic, shadowing, etc.) the drag force cannot be calculated in the general form—we leave this problem for future work.

For the fluid approach to be applicable, we naturally require that $R \gg n^{-1/3}$. Let us also assume a strong-friction case, viz.,

[a)]e-mail: ivlev@mpe.mpg.de.

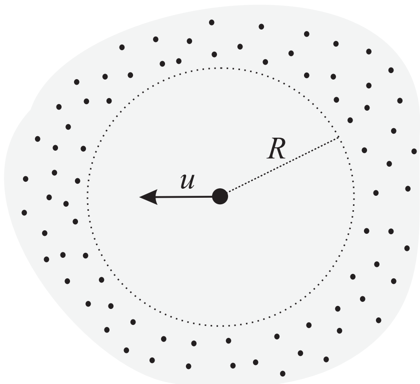

FIG. 1. Sketch showing a *subsonic projectile* (large bullet point in the center) moving with the velocity u in a *particle fluid* (small bullet points around), with a neutral gas as a still background exerting friction both on the projectile and on the particles. The projectile generates a spherical cavity of radius R (independent of u), from which particles are expelled due to repulsive interaction with the projectile. The physical size of the projectile is much smaller than R, the size of particles is much smaller than the mean interparticle distance.

$$\nu \gg \frac{u}{R}, \tag{3}$$

so that the second (nonlinear) term on the lhs of Eq. (1) can be neglected. Then, by taking the divergence of Eq. (1) and using Eq. (2), we obtain

$$\nabla^2 p = 0. \tag{4}$$

We immediately conclude that our problem is reduced to the textbook problem of a potential flow around a sphere,[22] with pressure playing the role of the velocity potential. Thus,

$$p(\mathbf{r}) = p_0 + c\frac{\mathbf{u}\cdot\mathbf{n}}{r^2}, \tag{5}$$

where $\mathbf{n} = \mathbf{r}/r$ is the radial unit vector and c is a constant. By substituting $p(\mathbf{r})$ in Eq. (1), omitting the nonlinear term, and using the boundary condition

$$(\mathbf{v}\cdot\mathbf{n})|_{r=R} = 0, \tag{6}$$

at the sphere surface, we get $c = -\frac{1}{2}\rho R^3\nu$. The resulting velocity

$$\mathbf{v}(\mathbf{r}) - \mathbf{u} = \frac{R^3}{2r^3}[\mathbf{u} - 3(\mathbf{u}\cdot\mathbf{n})\mathbf{n}], \tag{7}$$

exactly coincides with the velocity of a classical potential flow obtained from Eq. (1) with $\nu = 0$. Furthermore, by assuming $\nabla\times\mathbf{v} = 0$ in Eqs. (1) and (2), one can easily show that $\mathbf{v}(\mathbf{r})$ remains unaffected by friction at *arbitrary* ν.

There is, however, the essential distinction between the considered problem and the case of classical potential flow: While the latter is non-dissipative and hence a projectile moves freely through the fluid,[22] in the presence of friction the projectile experiences the Epstein drag force $F_{\rm Ep} = -m_{\rm p}\nu_{\rm p}u$. Moreover, one can immediately see that the dipolar pressure field [Eq. (5)] leads to an *additional* drag force

$$F_{\rm add} = 2\pi R^2\int_{-1}^{1} d\cos\theta\cos\theta\, p(R,\cos\theta), \tag{8}$$

where θ is the angle between $\mathbf{r}$ and $\mathbf{u}$. (Note that here we used the reciprocity of interactions between the projectile and particles.) This force is associated with the dissipation in the surrounding fluid (due to the particle friction against the neutral gas) and therefore can be considered as *indirect* neutral drag acting on the projectile. By substituting Eq. (5) in Eq. (8), we derive

$$F_{\rm add} = -\frac{2\pi}{3}\rho R^3\nu u, \tag{9}$$

and taking into account that $m\nu = (a/a_{\rm p})^2 m_{\rm p}\nu_{\rm p}$, we obtain the total drag force

$$F_{\rm drag} = F_{\rm Ep} + F_{\rm add} = -\left(1 + \frac{N}{2}\frac{a^2}{a_{\rm p}^2}\right)m_{\rm p}\nu_{\rm p}u, \tag{10}$$

where $N \equiv \frac{4\pi}{3}R^3 n \gg 1$ is the number of particles "excluded" from the effective sphere around the projectile.

For the experiment discussed in Ref. 21, where the projectile and particle diameters were $2a_{\rm p} = 15\ \mu$m and $2a = 2.55\ \mu$m, respectively, the effective projectile radius was estimated as $R \sim 3\times10^{-2}$ cm and the Wigner-Seitz radius of fluid particles was $(3/4\pi n)^{1/3} \simeq 9\times10^{-3}$ cm (so that $N \sim 30$). This yields $F_{\rm drag} \simeq -1.5 m_{\rm p}\nu_{\rm p}u$, i.e., the drag force on the projectile in this experiment is increased by about 50% due to the indirect gas friction. For the experiment illustrated in Fig. 3(a) of Ref. 16, the contribution of the indirect gas friction is presumably much larger: While the number of excluded particles N can be roughly estimated as several dozens, the ratio of the projectile to particle diameters is about two ($2a_{\rm p} = 20.0\ \mu$m and $2a = 9.55\ \mu$m), so that $F_{\rm add} \gg F_{\rm Ep}$ in this case. (We note that the projectile in this experiment moved at about a sound speed, so that the cavity around it was significantly distorted.)

We point out that the above consideration is also applicable for two-dimensional systems. By replacing n with the areal density $n_{\rm 2D}$, we derive $F_{\rm add,2D} = -\pi\rho_{\rm 2D}R^2\nu u$ for the additional drag, so that the total drag force is $F_{\rm drag,2D} = -[1 + N(a/a_{\rm p})^2]m_{\rm p}\nu_{\rm p}u$, where $N = \pi R^2 n_{\rm 2D}$ is the number of fluid particles excluded from the effective circular area.

III. VISCOUS FLOW

Now we consider a general case of viscous flow, when Eq. (1) is replaced with the modified Navier-Stokes equation

$$\nu(\mathbf{v}-\mathbf{u}) + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\frac{\nabla p}{\rho} + \frac{\eta}{\rho}\nabla^2\mathbf{v}, \tag{11}$$

where η is the dynamic shear viscosity of the particle fluid. Again, we shall neglect the nonlinear convective term on the rhs of Eq. (11), which requires the following condition to be satisfied:

$$\min\left\{\frac{u}{R\nu}, \frac{\rho u R}{\eta}\right\} \ll 1. \tag{12}$$

The non-viscous regime studied in Sec. II corresponds to the limit of infinite Reynolds numbers, $\rho u R/\eta \equiv Re \to \infty$, and Eq. (12) is then reduced to the strong-friction condition (3).

To solve the problem, we take the curl of Eq. (11) and obtain

$$\nu(\nabla\times\mathbf{v})=\frac{\eta}{\rho}\nabla^2(\nabla\times\mathbf{v}). \tag{13}$$

Following the standard route,[22] in order to satisfy Eq. (2) identically, we represent the velocity in the following form:

$$\mathbf{v}-\mathbf{u}=\nabla\times(\nabla F\times\mathbf{u}), \tag{14}$$

where $F(r)$ is an unknown scalar function. Since $\nabla F\times\mathbf{u}=\nabla\times(F\mathbf{u})$, we obtain

$$\nabla\times\mathbf{v}=\nabla\times\nabla\times\nabla\times(F\mathbf{u})=-\nabla^2\nabla\times(F\mathbf{u}).$$

Then Eq. (13) yields

$$\nabla(\nabla^2F-L_{\rm fr}^2\nabla^4F)=0, \tag{15}$$

where $L_{\rm fr}$ is the *friction length*

$$L_{\rm fr}=\sqrt{\frac{\eta}{\rho\nu}}.$$

Using condition (12), one can distinguish two limiting regimes: The *friction-dominated* regime corresponding to $L_{\rm fr}\ll R$, and the *viscosity-dominated* (Stokes) regime in the opposite limit. Taking into account that **v–u** should vanish at $r\to\infty$, we obtain the following solution of Eq. (15):

$$F(r)=2aL_{\rm fr}\left(1-\frac{1-e^{-r/L_{\rm fr}}}{r/L_{\rm fr}}\right)+\frac{b}{r}, \tag{16}$$

where a and b are constants (we wrote Eq. (16) in the form which is reduced to $ar+b/r$ in the Stokes limit). By substituting $F(r)$ in Eq. (14), we derive the velocity

$$\mathbf{v}(\mathbf{r})-\mathbf{u}=\left[\frac{3\tilde b}{x^3}-2\tilde a\frac{3-(3+3x+x^2)e^{-x}}{x^3}\right](\mathbf{u}\cdot\mathbf{n})\mathbf{n}-\left[\frac{\tilde b}{x^3}-2\tilde a\frac{1-(1+x+x^2)e^{-x}}{x^3}\right]\mathbf{u}, \tag{17}$$

where $\tilde a=a/L_{\rm fr}$, $\tilde b=b/L_{\rm fr}^3$, and $x=r/L_{\rm fr}$. In order to determine the constants, we have to impose proper boundary conditions at the surface of the effective sphere. Obviously, the classical no-slip condition normally employed for the Navier-Stokes equation cannot be used in our case. Instead, we assume that the tangential stress in the azimuthal direction, $\sigma_{r\theta}(\mathbf{r})$, vanishes at the surface. This condition

$$\left.\left(\frac{1}{r}\frac{\partial v_r}{\partial\theta}+\frac{\partial v_\theta}{\partial r}-\frac{v_\theta}{r}\right)\right|_{r=R}=0, \tag{18}$$

(written in the spherical coordinates) is complemented with the condition $v_r|_{r=R}=0$ [i.e., Eq. (6)]. By substituting Eq. (17) in Eqs. (6) and (18), we obtain the constants

$$\tilde a=\frac{3\tilde R e^{\tilde R}}{2(\tilde R+3)},$$

$$\tilde b=\frac{3\tilde R}{\tilde R+3}\left(e^{\tilde R}-1-\tilde R-\frac{1}{2}\tilde R^2-\frac{1}{6}\tilde R^3\right),$$

which are determined by the following parameter:

$$\tilde R=R/L_{\rm fr}. \tag{19}$$

With these constants, Eq. (17) is reduced to Eq. (7) in the friction-dominated limit ($\tilde R\to\infty$), while in the opposite viscosity-dominated limit ($\tilde R\to0$), we obtain

$$\mathbf{v}(\mathbf{r})-\mathbf{u}=-\frac{R}{2r}[\mathbf{u}+(\mathbf{u}\cdot\mathbf{n})\mathbf{n}]. \tag{20}$$

This velocity is different from the classical Stokes solution,[22] since we used Eq. (18) instead of the no-slip boundary condition.

Let us illustrate the flow patterns for different regimes. Figure 2 shows the streamlines calculated for (a) the friction-dominated limit, (b) an intermediate regime, and (c) the viscosity-dominated limit. One can see that the streamlines change very little, becoming somewhat smoother for smaller $\tilde R$. In Fig. 3, we plot the longitudinal velocity profiles in the transverse direction (calculated at $x=0$ and corresponding to the cases shown in Fig. 2). Here, the two limiting regimes are characterized by essentially different velocity distributions. It is noteworthy that the chosen intermediate regime

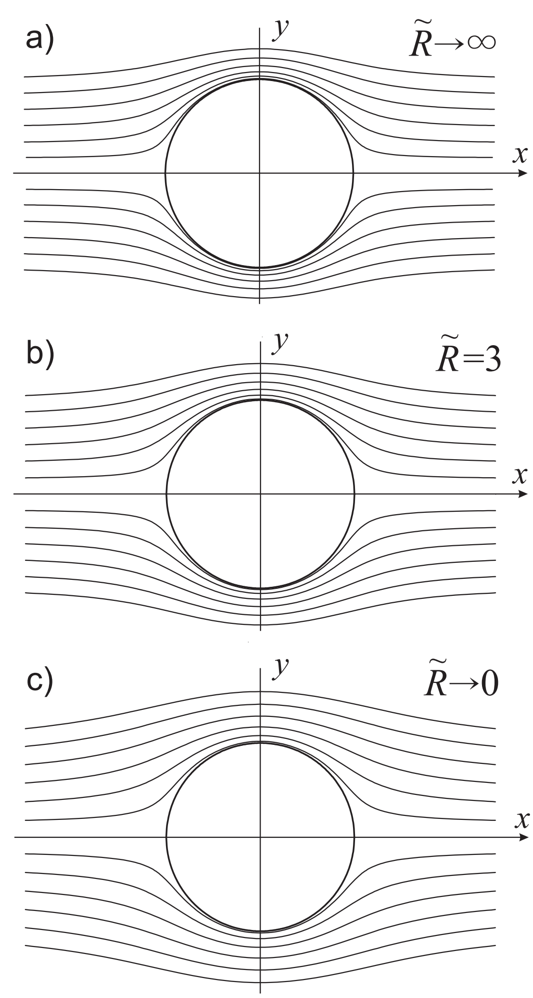

FIG. 2. Streamlines in different regimes. (a) Friction-dominated limit $\tilde R\to\infty$, Eq. (7); (b) intermediate regime $\tilde R=3$, Eq. (17); (c) viscosity-dominated limit $\tilde R\to0$, Eq. (20). Flow is along the x-axis.

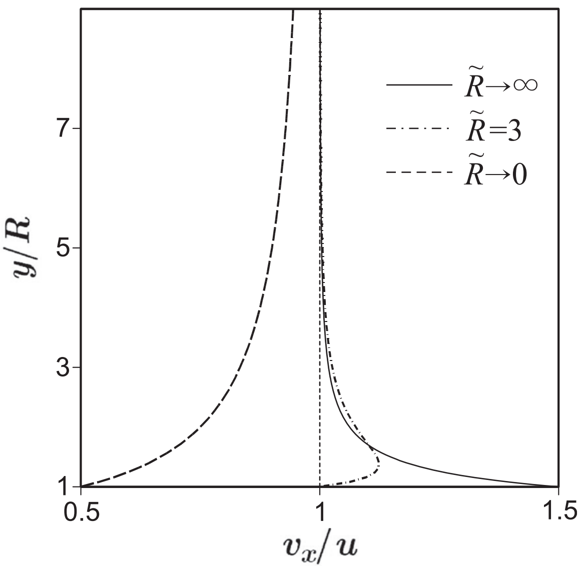

FIG. 3. Longitudinal velocity profiles in the transverse direction, $v_x(y)$, calculated for $x = 0$. The solid, dotted-dashed, and dashed lines correspond to the cases (a), (b), and (c), respectively, shown in Fig. 2. The longitudinal velocity is normalized by the flow velocity u, the transverse distance is normalized by the sphere radius R. The vertical dotted line indicates the common asymptote $v_x = u$.

(which approximately corresponds to the experiment of Ref. 21) has a non-monotonous velocity profile near the sphere, but its asymptotic behavior practically coincides with the friction-dominated case.

The pressure field can be derived from Eq. (11) (with omitted nonlinear term) by substituting Eq. (14) for the velocity, which readily gives us

$$p(\mathbf{r}) = p_0 - \rho\nu\mathbf{u}\cdot\nabla(F - L_{\rm fr}^2\nabla^2 F).$$

Since $\sigma_{r\theta}$ vanishes at the surface, one can directly use Eq. (8) to calculate the additional drag force on the projectile. Now, instead of the pressure we substitute the radial component of the stress tensor, $-\sigma_{rr} = p - 2\eta\,(\partial v_r/\partial r)$.[22] This yields the following general expression for the force:

$$F_{\rm add} = -4\pi\eta Ru\left(\frac{1+\tilde{R}+\frac{1}{6}\tilde{R}^2+\frac{1}{18}\tilde{R}^3}{1+\frac{1}{3}\tilde{R}}\right). \qquad (21)$$

In the friction-dominated regime, the resulting drag force naturally tends to Eq. (9) derived in Sec. II. In the viscosity-dominated limit, we obtain $F_{\rm add} = -4\pi\eta Ru$, which is 2/3 of the classical Stokes value—again, this is because Eq. (18) was employed as the boundary condition. We note that the use of (inappropriate here) no-slip boundary condition yields $F_{\rm add} = -6\pi\eta Ru(1+\tilde{R}+\frac{1}{9}\tilde{R}^2)$, and then the classical Stokes force is recovered for $\tilde{R}\to 0$ (while for the friction-dominated regime $F_{\rm add}$ remains unchanged).

For the experiment of Ref. 21, the friction length is estimated as $L_{\rm fr}\sim 10^{-2}$ cm. Using $R\sim 3\times 10^{-2}$ cm for the projectile radius, we conclude that viscosity contributes about (10–20)% to $F_{\rm add}$, i.e., the additional drag force for this experiment is friction-dominated. The role of viscosity should increase at lower pressures p, since $L_{\rm fr}\propto p^{-1/2}$ (while the dependence on the fluid number density n and particle size a is rather weak), and for smaller projectiles (provided the condition $R \gg n^{-1/3}$ is still satisfied).

IV. CONCLUSION

We derived a general velocity distribution of the flow induced in a fluid complex plasma by a moving projectile. Such flows generally depend on two similarity parameters: the Reynolds number Re and the parameter $\tilde{R}$ which characterizes the relative importance of the viscous and frictional dissipation mechanisms. The applicability condition of our results is presented by Eq. (12): If the first term in the curly braces dominates, i.e., when $\tilde{R}$ is small (viscosity-dominated regime), then Eq. (12) is reduced to the classical Stokes condition $Re \ll 1$; in the opposite case of large $\tilde{R}$ (friction-dominated regime) Eq. (12) is equivalent to the strong-friction condition (3).

Furthermore, we calculated total drag force on a projectile, and showed that it contains an additional part due to dissipation in the particle fluid. Depending on the value of $\tilde{R}$, one can distinguish between the friction-dominated and viscosity-dominated additional drag: The former is associated with the neutral gas friction on the surrounding particles (and provides additional contribution to the regular Epstein drag), the latter is caused by the viscous dissipation in the particle fluid. The analysis of available experiments suggests that the additional drag is normally comparable to, or even larger than the Epstein force and, hence, it can strongly affect the projectile dynamics.

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